

## Variability of the Period of the Star DU Monocerotis, an RR Lyrae Variable with the Blazhko Effect

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**Abstract**—In 2012–2014 we obtained 3641 CCD frames of the fields of the RR Lyrae (AB subtype,  $P = 0.583$  days) variable DU Mon with  $BVI_c$  filters using the 76-cm telescope of the South African Astronomical Observatory (SAAO) and the 1-m telescopes of the Las Cumbres Observatory Global Telescope Network (LCOGT). Our observations confirmed the presence of the Blazhko effect that we suspected previously and allowed its period to be determined,  $P_{Bl} = 60^d52 \pm 0^m03$ . Using all of the available observations, we constructed an O–C diagram spanning a time interval of 86 years that revealed at least one abrupt change in the pulsation period (a decrease by 15.26 s).

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### INTRODUCTION

While observing the star RW Dra, Blazhko detected periodic variations in the shape of its light curve (Blazhko 1907). This phenomenon was named the Blazhko effect after its discoverer. The pulsation periods of RR Lyrae stars are, on average, 0.5 days, while the Blazhko periods are longer by a factor of 10–1000, making it difficult to find and study this effect; therefore, the Blazhko effect has long been believed to be a rarity. However, recent special ground-based (Jurcsik et al. 2009; Le Borgne et al. 2012) and spaceborne (Szabó et al. 2009; Benkő et al. 2010) projects showed the fraction of RR Lyrae stars with the Blazhko effect to reach 50% of the investigated stars; whereas the Blazhko effect was previously associated almost exclusively with RRAB subtype stars, now, as the accuracy improves, it is also found for RRC subtype stars.

Although the number of known RR Lyrae stars with the Blazhko effect already exceeds 400 (405 by August 2016) (Skarka 2013; <http://physics.muni.cz/blasgalf/>), the nature of this phenomenon is still unknown. Therefore, searching for new such objects and investigating their properties, with the most important of them being the stability of the pulsation period (whose variations correlate with the variations of the Blazhko period), is important and topical.

The goal of this paper is to obtain new photometric observations to find the Blazhko period for the RR Lyrae star DU Mon, whose presence was suspected previously (Berdnikov et al. 2012), and to study the stability of its pulsation period.

### OBSERVATIONAL DATA

The CCD observations of DU Mon were performed during five observing seasons from December 2010 to May 2014 (JD 2455563–56785) with the 76-cm telescope of the South African Astronomical

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**Table 1.** CCD observations of DU Mon

HJD 2400000+	Filter	Magnitude	HJD 2400000+	Filter	Magnitude	HJD 2400000+	Filter	Magnitude
55896.47791	<i>B</i>	15.203	55896.47838	<i>V</i>	14.431	55896.47880	<i>I<sub>c</sub></i>	13.408
55898.52936	<i>B</i>	14.375	55898.52985	<i>V</i>	13.833	55898.53026	<i>I<sub>c</sub></i>	13.085
55899.42226	<i>B</i>	15.238	55899.42273	<i>V</i>	14.439	55899.42315	<i>I<sub>c</sub></i>	13.402
55900.39333	<i>B</i>	14.689	55900.39381	<i>V</i>	14.058	55900.39423	<i>I<sub>c</sub></i>	13.173
55901.40157	<i>B</i>	14.769	55901.40193	<i>V</i>	14.160	55901.40223	<i>I<sub>c</sub></i>	13.298
55901.52851	<i>B</i>	14.490	55901.52888	<i>V</i>	13.959	55901.52918	<i>I<sub>c</sub></i>	13.132
55902.42408	<i>B</i>	15.265	55902.42444	<i>V</i>	14.522	55902.42474	<i>I<sub>c</sub></i>	13.507
55902.52295	<i>B</i>	15.178	55902.52320	<i>V</i>	14.472	55902.52339	<i>I<sub>c</sub></i>	13.507
55903.38955	<i>B</i>	14.958	55903.38980	<i>V</i>	14.263	55903.39000	<i>I<sub>c</sub></i>	13.313
55904.39406	<i>B</i>	14.242	55904.39442	<i>V</i>	13.712	55904.39472	<i>I<sub>c</sub></i>	13.015
55904.50382	<i>B</i>	14.794	55904.50418	<i>V</i>	14.177	55904.50449	<i>I<sub>c</sub></i>	13.255
55905.38768	<i>B</i>	15.273	55905.38792	<i>V</i>	14.555	55905.38810	<i>I<sub>c</sub></i>	13.519
55905.50296	<i>B</i>	14.647	55905.50320	<i>V</i>	14.031	55905.50339	<i>I<sub>c</sub></i>	13.201
55905.51642	<i>B</i>	14.401	55905.51666	<i>V</i>	13.854	55905.51686	<i>I<sub>c</sub></i>	13.108
55905.52881	<i>B</i>	14.206	55905.52907	<i>V</i>	13.690	55905.52925	<i>I<sub>c</sub></i>	12.992
55905.53738	<i>B</i>	14.152	55905.53762	<i>V</i>	13.657	55905.53782	<i>I<sub>c</sub></i>	12.970
55905.54797	<i>B</i>	14.125	55905.54821	<i>V</i>	13.632	55905.54840	<i>I<sub>c</sub></i>	12.957
55906.42805	<i>B</i>	15.222	55906.42841	<i>V</i>	14.493	55906.42871	<i>I<sub>c</sub></i>	13.489
55906.51667	<i>B</i>	15.320	55906.51703	<i>V</i>	14.539	55906.51733	<i>I<sub>c</sub></i>	13.508
55906.53412	<i>B</i>	15.329	55906.53453	<i>V</i>	14.535	55906.53479	<i>I<sub>c</sub></i>	13.519
55906.58590	<i>B</i>	15.287	55906.58643	<i>V</i>	14.528	55906.58674	<i>I<sub>c</sub></i>	13.567
55907.38334	<i>B</i>	14.612	55907.38359	<i>V</i>	14.007	55907.38378	<i>I<sub>c</sub></i>	13.147
55907.39407	<i>B</i>	14.652	55907.39431	<i>V</i>	14.010	55907.39449	<i>I<sub>c</sub></i>	13.143
55907.41841	<i>B</i>	14.754	55907.41865	<i>V</i>	14.111	55907.41884	<i>I<sub>c</sub></i>	13.204
55907.44248	<i>B</i>	14.858	55907.44284	<i>V</i>	14.141	55907.44308	<i>I<sub>c</sub></i>	13.232
55907.45560	<i>B</i>	14.891	55907.45601	<i>V</i>	14.207	55907.45626	<i>I<sub>c</sub></i>	13.278
55908.38813	<i>B</i>	15.075	55908.38849	<i>V</i>	14.362	55908.38879	<i>I<sub>c</sub></i>	13.442
55908.39349	<i>B</i>	15.003	55908.39385	<i>V</i>	14.289	55908.39415	<i>I<sub>c</sub></i>	13.401
55908.40165	<i>B</i>	14.923	55908.40202	<i>V</i>	14.239	55908.40232	<i>I<sub>c</sub></i>	13.317
55908.41033	<i>B</i>	14.749	55908.41057	<i>V</i>	14.105	55908.41077	<i>I<sub>c</sub></i>	13.291
55908.41442	<i>B</i>	14.700	55908.41520	<i>V</i>	14.075	55908.41579	<i>I<sub>c</sub></i>	13.248
55908.44552	<i>B</i>	14.274	55908.44624	<i>V</i>	13.732	55908.44683	<i>I<sub>c</sub></i>	13.015
55908.44753	<i>B</i>	14.239	55908.44823	<i>V</i>	13.688	55908.44882	<i>I<sub>c</sub></i>	12.967
55908.44951	<i>B</i>	14.236	55908.45021	<i>V</i>	13.694	55908.45080	<i>I<sub>c</sub></i>	12.966
55908.45147	<i>B</i>	14.214	55908.45218	<i>V</i>	13.708	55908.45277	<i>I<sub>c</sub></i>	12.991
55908.45370	<i>B</i>	14.190	55908.45481	<i>V</i>	13.576	55908.45683	<i>B</i>	14.211
55910.46914	<i>V</i>	14.407	55910.46950	<i>I<sub>c</sub></i>	13.403	55911.34177	<i>V</i>	14.003
55911.34201	<i>I<sub>c</sub></i>	13.181	55911.37928	<i>B</i>	14.090	55911.37953	<i>V</i>	13.588

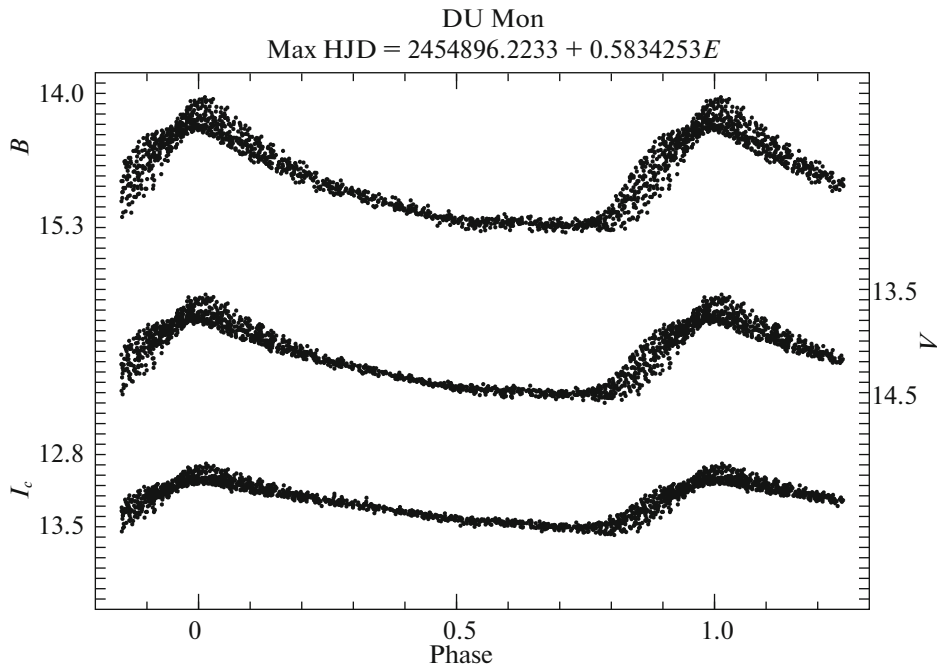


Fig. 1. Phase light curves of the RR Lyrae star DU Mon in the  $BVI_c$  bands. The large scatter is caused by the Blazhko effect.

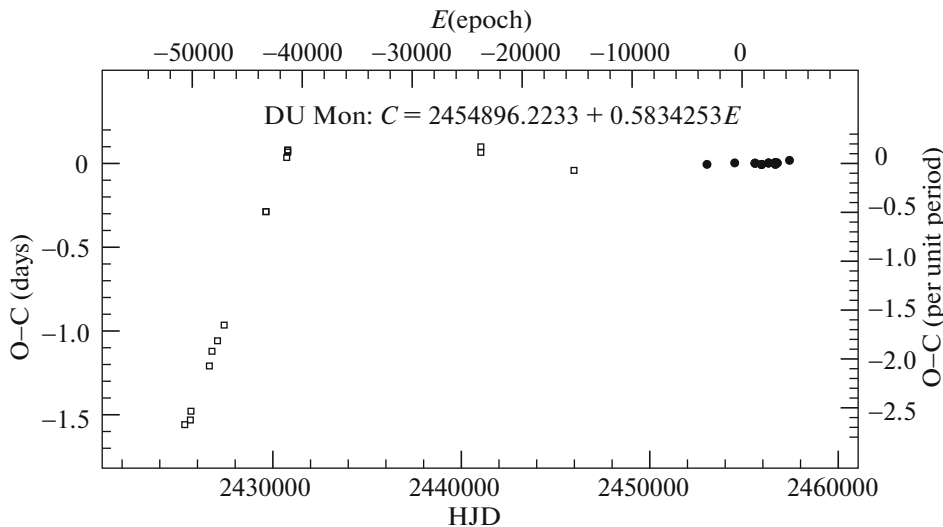


Fig. 2. O–C diagram for the RR Lyrae star DU Mon.

Observatory (SAAO) in the Republic of South Africa, where an SBIG ST-10XME CCD camera equipped with Kron–Cousins  $BVI_c$  filters (Cousins 1976) was used. We obtained a total of 3139 CCD frames; the photometric errors are close to  $0^m.01$ . The observing and data reduction techniques are described in our previous paper (Berdnikov et al. 2012), where we presented the results of our observations in the

first season (370 CCD frames from December 2010 to January 2011) and suspected the presence of the Blazhko effect for several RR Lyrae stars, including DU Mon.

In addition, we used the Las Cumbres Observatory Global Telescope (LCOGT) Network (Brown et al. 2013), where 872 CCD frames with  $BVI_c$  filters were obtained in the interval JD 2456634–56688. To

**Table 2.** Observational data for DU Mon

Data source	Number of observations	Type of observations	Interval of Julian dates
Ahnert et al. (1949)	13	Photographic brightening (PB)	2425324–2430790
This paper	3	Photographic brightening (PB)	2441035–2445978
ASAS-3 (Pojmanski 2002)	219	CCD ( <i>V</i> )	2451868–2453818
Berdnikov et al. (2012)	126	CCD ( <i>B</i> )	2455563–2455593
Berdnikov et al. (2012)	123	CCD ( <i>V</i> )	2455563–2455593
Berdnikov et al. (2012)	121	CCD ( <i>I<sub>c</sub></i> )	2455563–2455593
This paper	1213	CCD ( <i>B</i> )	2455896–2456785
This paper	1237	CCD ( <i>V</i> )	2455896–2456785
This paper	1191	CCD ( <i>I<sub>c</sub></i> )	2455896–2456785
AAVSO	334	CCD ( <i>V</i> )	2457410–2457415

reduce the LCOGT observations, we applied differential photometry relative to the secondary standards whose magnitudes were determined from the SAAO observations; the photometric errors here are close to 0<sup>m</sup>.02.

All our unpublished observations are presented in Table 1 (which contains 3641 magnitude measurements and is fully given in electronic form at the CDS portal <ftp://cdsarc.u-strasbg.fr/pub/cats/J/PAZh/>) and are shown graphically in Fig. 1.

In addition to our observations, we used the CCD observations from the ASAS-3 catalogue (Pojmanski 2002) and the International Database of the American Association of Variable Star Observers (AAVSO) as well as the times of brightenings of DU Mon found on photographic plates from the collections of the Sternberg Astronomical Institute and the Sonneberg Observatory (Ahnert et al. 1949).

Information about the number of observations used is presented in Table 2, according to which the data used span a time interval of 86 years.

## VARIABILITY OF THE PULSATION PERIOD

To study the variability of the pulsation period of DU Mon, we use the universally accepted technique of analyzing the O–C diagrams. The most accurate method of determining the O–C residuals is the method by Hertzsprung (1919), whose computer implementation is described in Berdnikov (1992).

The results of our reduction of the seasonal light curves for DU Mon are presented in Table 3: the first and second columns contain the epochs of maximum brightness and their errors, the third column gives the type of observations used (see Table 2), the fourth and fifth columns give the epoch number *E* and the O–C residual, and the sixth and seventh columns give the number of observations *N* and the data source. The data obtained by Hertzsprung’s method are indicated on the O–C diagram (Fig. 2) by the circles with vertical bars that specify the error limits of the O–C residuals; the squares are the times of photographic brightenings.

The O–C diagram can be represented by segments of straight lines, which points to an abrupt decrease in the period near JD 2431000 ( $\delta P =$

**Table 3.** Epochs of maximum brightness for DU Mon

Epoch of maximum (HJD)	Error, days	Band	$E$	O–C, days	$N$	Data source
2425324.3360	–	PB	–50684	–1.5591	1	Ahnert et al. (1949)
2425622.4940	–	PB	–50173	–1.5314	1	Ahnert et al. (1949)
2425653.4680	–	PB	–50120	–1.4789	1	Ahnert et al. (1949)
2426631.5590	–	PB	–48444	–1.2087	1	Ahnert et al. (1949)
2426769.3350	–	PB	–48208	–1.1211	1	Ahnert et al. (1949)
2427063.4440	–	PB	–47704	–1.0585	1	Ahnert et al. (1949)
2427416.5100	–	PB	–47099	–0.9648	1	Ahnert et al. (1949)
2429633.6200	–	PB	–43300	–0.2875	1	Ahnert et al. (1949)
2429635.3700	–	PB	–43297	–0.2877	1	Ahnert et al. (1949)
2430735.4500	–	PB	–41412	0.0356	1	Ahnert et al. (1949)
2430784.5000	–	PB	–41328	0.0778	1	Ahnert et al. (1949)
2430787.4200	–	PB	–41323	0.0807	1	Ahnert et al. (1949)
2430790.3240	–	PB	–41318	0.0676	1	Ahnert et al. (1949)
2441035.2710	–	PB	–23758	0.0663	1	This paper
2441035.3040	–	PB	–23758	0.0993	1	This paper
2445978.5260	–	PB	–15285	–0.0413	1	This paper
2453034.5077	0.0102	$V$	–3191	–0.0054	109	Pojmanski (2002)
2454505.9151	0.0059	$V$	–669	0.0033	110	Pojmanski (2002)
2455584.0822	0.0007	$B$	1179	0.0008	126	Berdnikov et al. (2012)
2455584.0829	0.0008	$V$	1179	0.0012	123	Berdnikov et al. (2012)
2455584.0860	0.0008	$I_c$	1179	0.0021	121	Berdnikov et al. (2012)
2455930.0474	0.0008	$V$	1772	–0.0055	418	This paper
2455931.2160	0.0009	$I_c$	1774	–0.0060	381	This paper
2455932.3797	0.0008	$B$	1776	–0.0066	380	This paper
2456300.5306	0.0009	$B$	2407	0.0029	277	This paper
2456300.5307	0.0009	$V$	2407	0.0028	275	This paper
2456300.5331	0.0010	$I_c$	2407	0.0029	272	This paper
2456652.9212	0.0010	$B$	3011	0.0046	222	This paper
2456652.9217	0.0010	$V$	3011	0.0048	222	This paper
2456652.9233	0.0011	$I_c$	3011	0.0042	221	This paper
2456662.2471	0.0008	$B$	3027	–0.0043	295	This paper
2456662.2474	0.0008	$V$	3027	–0.0043	283	This paper
2456662.2491	0.0009	$I_c$	3027	–0.0048	278	This paper
2456756.7681	0.0021	$V$	3189	0.0015	39	This paper
2456756.7697	0.0021	$B$	3189	0.0035	39	This paper
2456756.7719	0.0023	$I_c$	3189	0.0031	39	This paper
2457413.1386	0.0010	$V$	4314	0.0186	334	AAVSO

**Table 4.** Ephemeris of DU Mon

Interval of Julian dates	$T_0$ (HJD)	Period, days
2425000–2431000	$2428057.9003 \pm 0.0053$	$0.58360189 \pm 0.00000144$
2453000–2457000	$2454896.2233 \pm 0.0016$	$0.58342530 \pm 0.00000064$

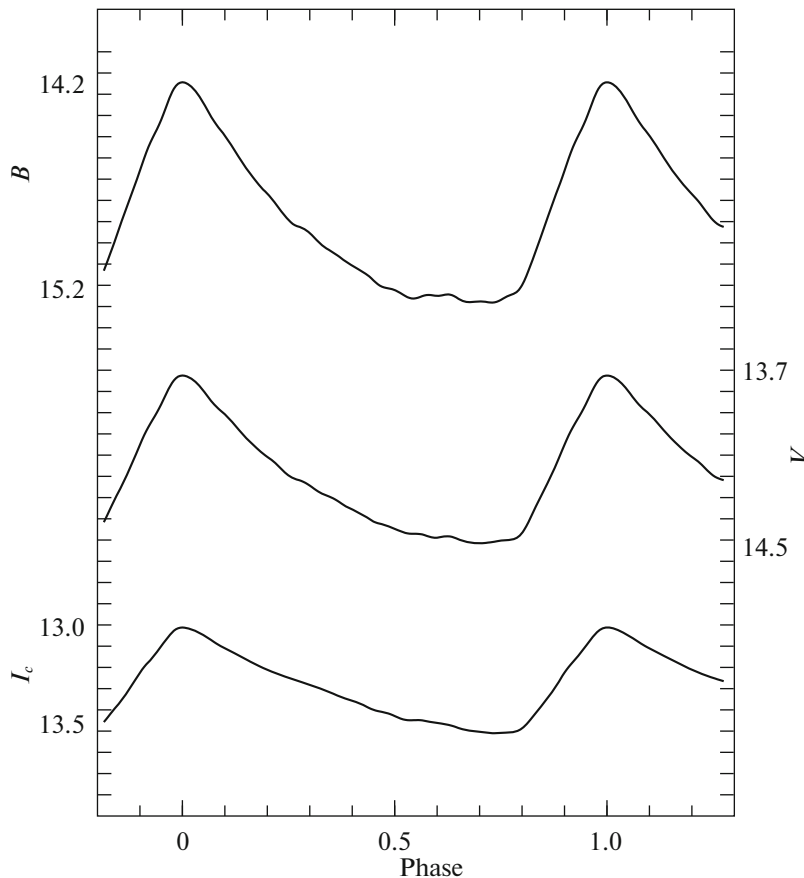


Fig. 3. Standard light curves of the RR Lyrae star DU Mon in the  $B$ ,  $V$ , and  $I_c$  bands.

–15.26 s). Table 4 gives the light elements before and after this date. There are big gaps on the diagram, where epoch miscalculations are possible.

Figure 2 shows that the pulsation period remained constant during our observations; therefore, we may use all our observations to determine the Blazhko period.

From our CCD observations we found that the brightness maxima in the  $B$  band occur earlier by  $0^{\text{d}}0003$  than the corresponding maxima in the  $V$  band, while the maxima in the  $I_c$  band occur later by  $0^{\text{d}}0009$  than that in the  $V$  band. These corrections were taken into account when constructing Fig. 2 and determining the light elements (Table 4), which, thus, refer to the  $V$  system. The O–C residuals in Fig. 2 and Table 3 were calculated with the current light elements (the bottom row in Table 4).

It should be noted that the results on the period variability obtained here are based on specific standard curves. Therefore, we present them in Table 5 (given fully in electronic form at the CDS portal

<ftp://cdsarc.u-strasbg.fr/pub/cats/J/PAZh/>) to be used in future studies and to establish the relationship to our data if other standard curves will be used. Table 5 contains the  $B$ ,  $V$ , and  $I_c$  magnitudes of DU Mon for phases from 0 to 0.995 with a 0.005 step; these standard curves displayed graphically in Fig. 3 were constructed from our CCD observations (Table 1).

## THE BLAZHKO EFFECT

To find the Blazhko period, we used the method by Goranskij (1976) that is as follows. For each trial Blazhko period  $P_{Bl}$  all observations are divided by phases of  $P_{Bl}$  into (in our case) 10 equal intervals within which they are sorted by phases of the fundamental period. Then, the total scatter parameter  $S_N$  is calculated, which is a normalized sum of the squares of the deviations of each succeeding point on the combined light curve from the preceding (in phase) point. For the real value of  $P_{Bl}$  in each of the 10 intervals (Fig. 4) there is almost no scatter of observations caused by the Blazhko effect itself;

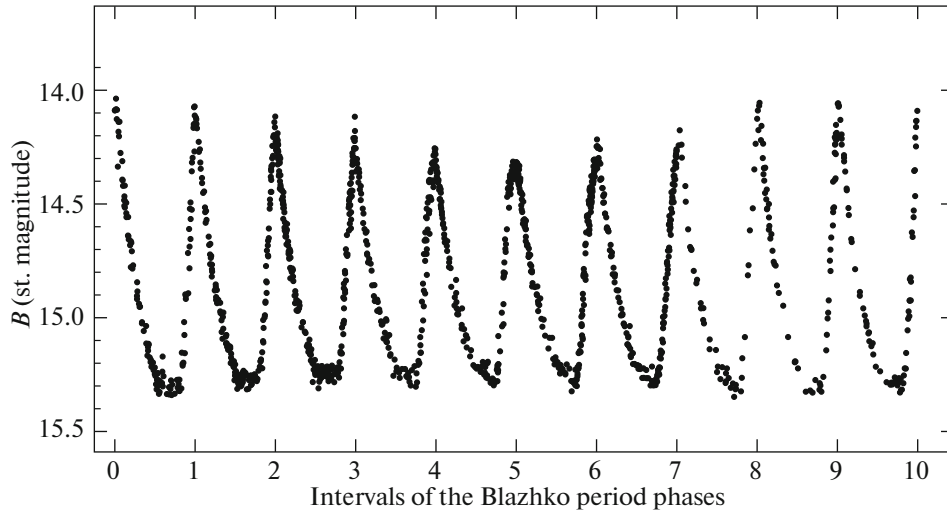


Fig. 4. Phase  $B$ -band light curve of DU Mon in ten consecutive narrow phase intervals of the Blazhko period ( $P_{Bl} = 60^{\text{d}}52$ ).

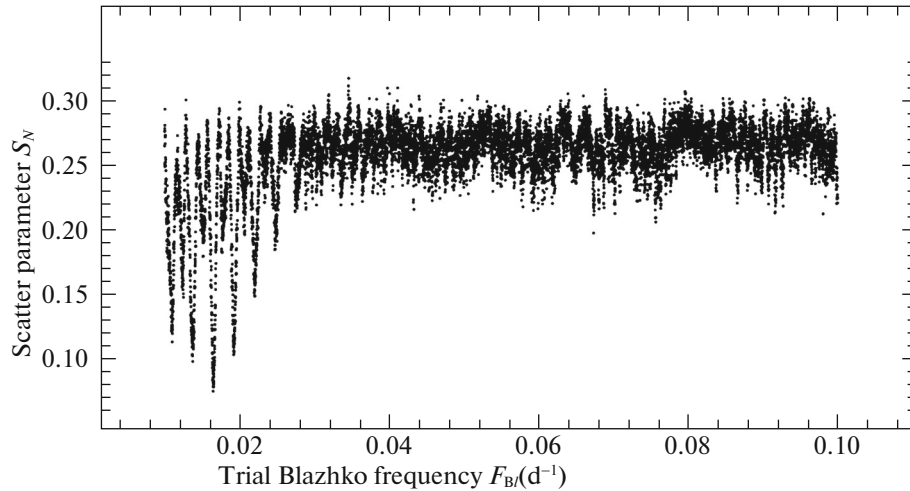


Fig. 5. Normalized scatter parameter  $S_N$  versus trial Blazhko frequency  $F_{Bl}$  for the  $B$ -band observations of DU Mon. The scatter parameter  $S_N$  reaches its minimum at  $F_{Bl} = 0.01652$  ( $P_{Bl} = 60^{\text{d}}53$ ).

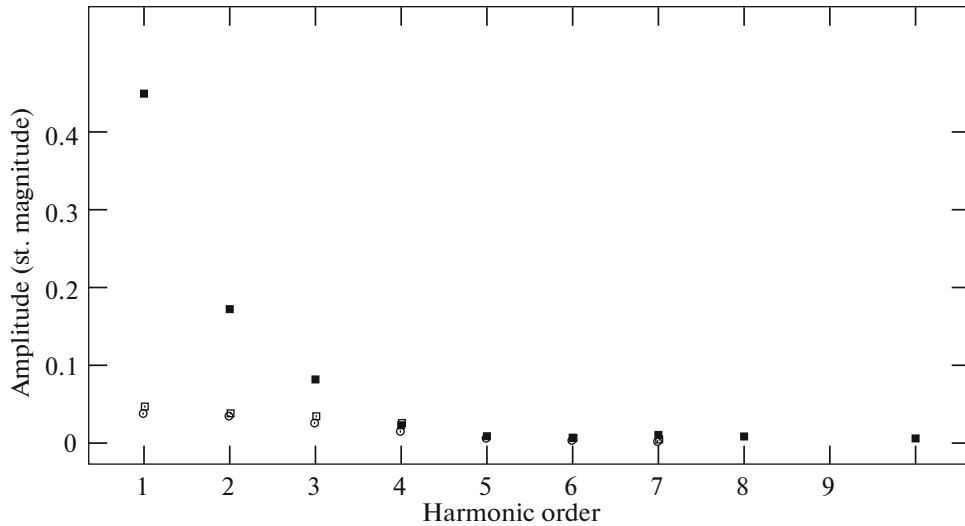
therefore, the scatter parameter  $S_N$  will be minimal (Fig. 5).

Unfortunately, the low accuracy of CCD observations from the ASAS-3 catalogue (where DU Mon is close to the limit; that is why the Blazhko effect was not detected by Szczygiel and Fabrycky (2007) who studied all RR Lyrae stars from the ASAS-3 catalogue) does not allow  $P_{Bl}$  to be determined. Therefore, we had to restrict ourselves only to our CCD observations in the  $B$ ,  $V$ , and  $I_c$  bands.

The minimum scatter parameter for 1339  $B$ -band observations (Fig. 5) corresponds to  $P_{Bl} =$

$60^{\text{d}}53$  ( $f_{Bl} = 0.01652 \text{ d}^{-1}$ ). For 1360  $V$  and 1312  $I_c$  observations the Blazhko period is close to  $60^{\text{d}}53$  ( $f_{Bl} = 0.01652 \text{ d}^{-1}$ ) and  $60^{\text{d}}50$  ( $f_{Bl} = 0.01653 \text{ d}^{-1}$ ), respectively.

Using the PERIOD04 code (Lenz and Breger 2005), we computed the power spectrum for the observations in three bands. Table 6 presents the most reliable frequencies for the triplets of harmonics of the pulsation period; the frequency difference between the components and the corresponding harmonic gives the Blazhko frequency. The dependence of the triplet amplitudes on the harmonic number shown in Fig. 6 is consistent with the results obtained



**Fig. 6.** *B*-band amplitudes from Table 6 versus harmonic number. The amplitudes of the harmonics, the left and right components of the triplets are indicated by the filled squares, circles, and open squares, respectively.

for other RR Lyrae stars with the Blazhko effect (Kolenberg et al. 2010; Kolenberg 2012). Kovács (2002) proposed to divide the RR Lyrae variables with

the Blazhko effect into classes with one component  $f+$  (BL1) and two symmetric components  $f- f+$  (BL2). As can be seen from Table 6, DU Mon belongs to the second class, although it should be noted that the triplet components are not symmetric.

**Table 5.** Standard light curves of DU Mon in the *B*, *V*, and *I<sub>c</sub>* bands (a fragment; given fully in electronic form)

Phase	<i>B</i>	<i>V</i>	<i>I<sub>c</sub></i>
0.000	14.244	13.725	13.012
0.005	14.245	13.726	13.013
0.010	14.249	13.729	13.014
0.015	14.255	13.733	13.017
0.020	14.262	13.739	13.020
0.025	14.272	13.745	13.024
0.030	14.283	13.754	13.028
0.035	14.296	13.763	13.032
0.040	14.311	13.774	13.037
0.045	14.327	13.785	13.042
0.050	14.344	13.798	13.048
0.055	14.361	13.811	13.054
0.060	14.379	13.825	13.061
0.065	14.396	13.838	13.068
0.070	14.413	13.850	13.074
0.075	14.428	13.861	13.081
0.080	14.443	13.872	13.087
0.085	14.457	13.881	13.094
0.090	14.470	13.890	13.099
0.095	14.483	13.899	13.105
0.100	14.496	13.908	13.110

Fifteen left and 14 right components of the triplets of the first five harmonics give a Blazhko frequency of 0.01660 and 0.01652 d<sup>-1</sup>, respectively, corresponding to periods of 60<sup>d</sup>.24 and 60<sup>d</sup>.53. The latter value is close to the results obtained by Goranskij's method; therefore, we discarded the first value and found the mean Blazhko period by the two methods:  $P_{Bl} = 60^d.52 \pm 0^d.03$ .

Figure 7 shows the light curves of DU Mon in the *B*, *V*, and *I<sub>c</sub>* bands folded with the pulsation period in the narrow phase intervals of  $P_{Bl}$  corresponding to the maximum and minimum amplitudes.

## CONCLUSIONS

(1) We obtained 3641 CCD frames in the *BVI<sub>c</sub>* system for the RR Lyrae star DU Mon. Our observations have allowed the Blazhko effect for this star to be detected for the first time and to determine its period:  $P_{Bl} = 60^d.52 \pm 0^d.03$ .

(2) We constructed the O–C diagram for DU Mon spanning a time interval of 86 years, which has allowed at least one abrupt change in the pulsation period near JD 2431000 to be detected for the first time.

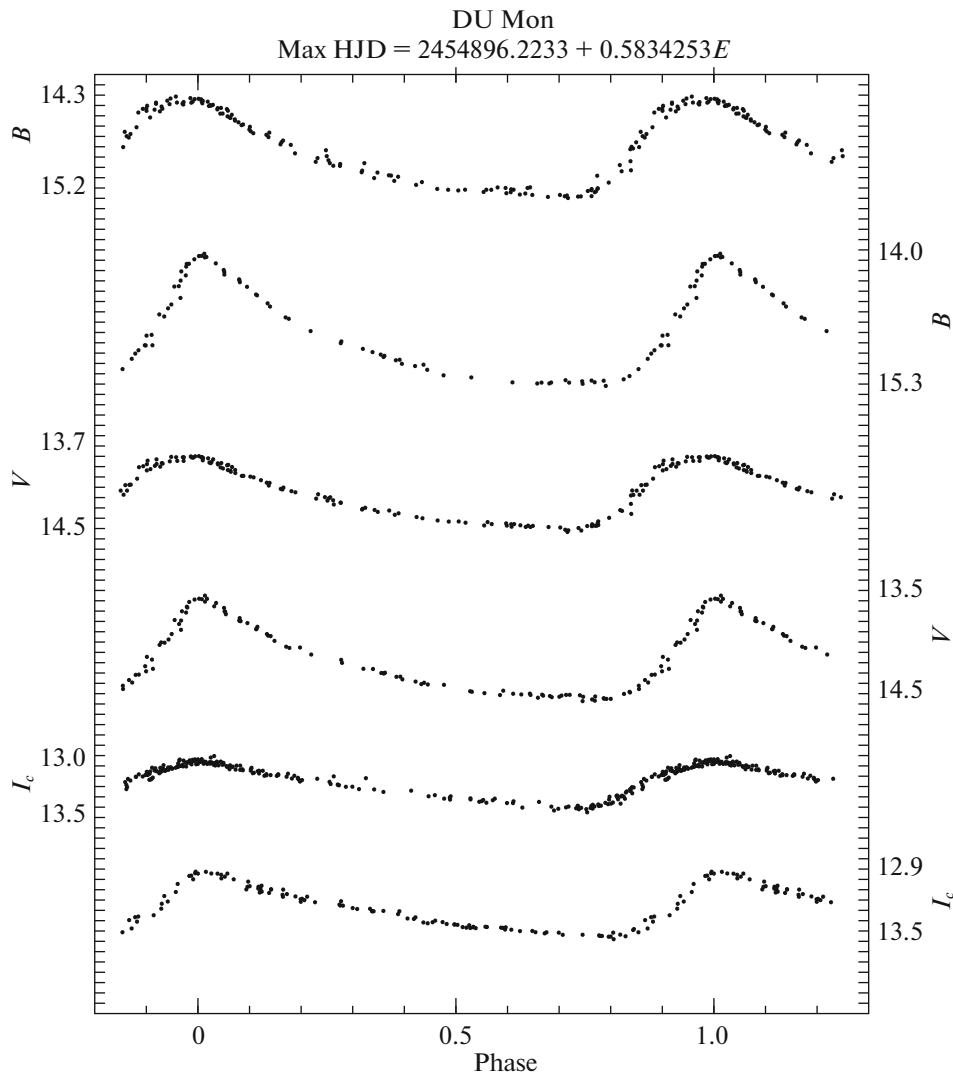


**Table 6.** Amplitudes and phases of the components of the pulsation and modulation oscillations in DU Mon

Filter	Frequency name	Frequency, d <sup>-1</sup>	Amplitude, mag	Phase, rad/2 $\pi$
<i>B</i>	$f_0 - f_{Bl}$	$1.6973621 \pm 0.0000179$	$0.0380 \pm 0.0015$	$0.7288 \pm 0.0063$
<i>V</i>	$f_0 - f_{Bl}$	$1.6973774 \pm 0.0000191$	$0.0273 \pm 0.0011$	$0.8736 \pm 0.0067$
<i>I<sub>c</sub></i>	$f_0 - f_{Bl}$	$1.6973790 \pm 0.0000215$	$0.0182 \pm 0.0009$	$0.7840 \pm 0.0076$
<i>V</i>	$f_0$	$1.7140153 \pm 0.0000015$	$0.3344 \pm 0.0011$	$0.7055 \pm 0.0005$
<i>B</i>	$f_0$	$1.7140156 \pm 0.0000015$	$0.4492 \pm 0.0015$	$0.6974 \pm 0.0005$
<i>I<sub>c</sub></i>	$f_0$	$1.7140157 \pm 0.0000018$	$0.2093 \pm 0.0009$	$0.6504 \pm 0.0006$
<i>V</i>	$f_0 + f_{Bl}$	$1.7305021 \pm 0.0000141$	$0.0368 \pm 0.0011$	$0.7487 \pm 0.0050$
<i>B</i>	$f_0 + f_{Bl}$	$1.7305185 \pm 0.0000145$	$0.0470 \pm 0.0015$	$0.8228 \pm 0.0051$
<i>I<sub>c</sub></i>	$f_0 + f_{Bl}$	$1.7305283 \pm 0.0000168$	$0.0233 \pm 0.0009$	$0.2741 \pm 0.0059$
<i>V</i>	$2f_0 - f_{Bl}$	$3.4114070 \pm 0.0000216$	$0.0241 \pm 0.0011$	$0.1311 \pm 0.0076$
<i>I<sub>c</sub></i>	$2f_0 - f_{Bl}$	$3.4114301 \pm 0.0000259$	$0.0151 \pm 0.0009$	$0.8504 \pm 0.0091$
<i>B</i>	$2f_0 - f_{Bl}$	$3.4114521 \pm 0.0000196$	$0.0347 \pm 0.0015$	$0.5869 \pm 0.0069$
<i>V</i>	$2f_0$	$3.4280290 \pm 0.0000039$	$0.1328 \pm 0.0011$	$0.8823 \pm 0.0013$
<i>I<sub>c</sub></i>	$2f_0$	$3.4280346 \pm 0.0000047$	$0.0821 \pm 0.0009$	$0.5523 \pm 0.0016$
<i>B</i>	$2f_0$	$3.4280418 \pm 0.0000039$	$0.1722 \pm 0.0015$	$0.1602 \pm 0.0014$
<i>V</i>	$2f_0 + f_{Bl}$	$3.4445624 \pm 0.0000182$	$0.0285 \pm 0.0011$	$0.2628 \pm 0.0064$
<i>B</i>	$2f_0 + f_{Bl}$	$3.4445726 \pm 0.0000178$	$0.0383 \pm 0.0015$	$0.6738 \pm 0.0062$
<i>I<sub>c</sub></i>	$2f_0 + f_{Bl}$	$3.4445762 \pm 0.0000214$	$0.0183 \pm 0.0009$	$0.4832 \pm 0.0075$
<i>I<sub>c</sub></i>	$3f_0 - f_{Bl}$	$5.1254344 \pm 0.0000315$	$0.0124 \pm 0.0009$	$0.6084 \pm 0.0111$
<i>V</i>	$3f_0 - f_{Bl}$	$5.1254655 \pm 0.0000270$	$0.0193 \pm 0.0011$	$0.8365 \pm 0.0095$
<i>B</i>	$3f_0 - f_{Bl}$	$5.1255145 \pm 0.0000264$	$0.0258 \pm 0.0015$	$0.0707 \pm 0.0093$
<i>V</i>	$3f_0$	$5.1420474 \pm 0.0000081$	$0.0641 \pm 0.0011$	$0.8006 \pm 0.0028$
<i>I<sub>c</sub></i>	$3f_0$	$5.1420480 \pm 0.0000091$	$0.0432 \pm 0.0009$	$0.7679 \pm 0.0032$
<i>B</i>	$3f_0$	$5.1420503 \pm 0.0000083$	$0.0818 \pm 0.0015$	$0.6342 \pm 0.0029$
<i>B</i>	$3f_0 + f_{Bl}$	$5.1585659 \pm 0.0000197$	$0.0345 \pm 0.0015$	$0.0164 \pm 0.0069$
<i>V</i>	$3f_0 + f_{Bl}$	$5.1585798 \pm 0.0000205$	$0.0253 \pm 0.0011$	$0.2396 \pm 0.0072$
<i>I<sub>c</sub></i>	$3f_0 + f_{Bl}$	$5.1585862 \pm 0.0000256$	$0.0153 \pm 0.0009$	$0.9026 \pm 0.0090$
<i>B</i>	$4f_0 - f_{Bl}$	$6.8394795 \pm 0.0000454$	$0.0150 \pm 0.0015$	$0.0017 \pm 0.0160$
<i>V</i>	$4f_0 - f_{Bl}$	$6.8394819 \pm 0.0000555$	$0.0093 \pm 0.0011$	$0.8705 \pm 0.0196$
<i>I<sub>c</sub></i>	$4f_0 - f_{Bl}$	$6.8395082 \pm 0.0000610$	$0.0064 \pm 0.0009$	$0.4354 \pm 0.0215$
<i>I<sub>c</sub></i>	$4f_0$	$6.8560302 \pm 0.0000305$	$0.0128 \pm 0.0009$	$0.7084 \pm 0.0107$
<i>V</i>	$4f_0$	$6.8560549 \pm 0.0000260$	$0.0200 \pm 0.0011$	$0.3144 \pm 0.0092$
<i>B</i>	$4f_0$	$6.8560610 \pm 0.0000297$	$0.0229 \pm 0.0015$	$0.9706 \pm 0.0105$

**Table 6.** (Contd.)

Filter	Frequency name	Frequency, d <sup>-1</sup>	Amplitude, mag	Phase, rad/2 $\pi$
<i>V</i>	$4f_0 + f_{Bl}$	$6.8725845 \pm 0.0000318$	$0.0164 \pm 0.0011$	$0.9820 \pm 0.0112$
<i>I<sub>c</sub></i>	$4f_0 + f_{Bl}$	$6.8725969 \pm 0.0000413$	$0.0095 \pm 0.0009$	$0.2947 \pm 0.0145$
<i>B</i>	$4f_0 + f_{Bl}$	$6.8726122 \pm 0.0000264$	$0.0258 \pm 0.0015$	$0.4309 \pm 0.0093$
<i>I<sub>c</sub></i>	$5f_0 - f_{Bl}$	$8.5533232 \pm 0.0000997$	$0.0039 \pm 0.0009$	$0.7614 \pm 0.0351$
<i>B</i>	$5f_0 - f_{Bl}$	$8.5534806 \pm 0.0001091$	$0.0062 \pm 0.0015$	$0.0198 \pm 0.0384$
<i>V</i>	$5f_0 - f_{Bl}$	$8.5535062 \pm 0.0000855$	$0.0061 \pm 0.0011$	$0.5111 \pm 0.0301$
<i>I<sub>c</sub></i>	$5f_0$	$8.5699773 \pm 0.0000752$	$0.0052 \pm 0.0009$	$0.5205 \pm 0.0265$
<i>B</i>	$5f_0$	$8.5700241 \pm 0.0000771$	$0.0088 \pm 0.0015$	$0.9095 \pm 0.0272$
<i>V</i>	$5f_0$	$8.5700815 \pm 0.0000685$	$0.0076 \pm 0.0011$	$0.6673 \pm 0.0241$
<i>I<sub>c</sub></i>	$5f_0 + f_{Bl}$	$8.5864100 \pm 0.0000649$	$0.0060 \pm 0.0009$	$0.7486 \pm 0.0229$
<i>V</i>	$5f_0 + f_{Bl}$	$8.5865444 \pm 0.0000644$	$0.0080 \pm 0.0011$	$0.2177 \pm 0.0227$
<i>B</i>	$6f_0 - f_{Bl}$	$10.2675857 \pm 0.0001960$	$0.0034 \pm 0.0015$	$0.0128 \pm 0.0691$
<i>B</i>	$6f_0$	$10.2839964 \pm 0.0000965$	$0.0070 \pm 0.0015$	$0.2482 \pm 0.0340$
<i>V</i>	$6f_0$	$10.2841505 \pm 0.0000802$	$0.0065 \pm 0.0011$	$0.5667 \pm 0.0283$
<i>I<sub>c</sub></i>	$6f_0$	$10.2842563 \pm 0.0001514$	$0.0026 \pm 0.0009$	$0.6511 \pm 0.0534$
<i>B</i>	$6f_0 + f_{Bl}$	$10.3003611 \pm 0.0001057$	$0.0064 \pm 0.0015$	$0.4624 \pm 0.0372$
<i>B</i>	$7f_0 - f_{Bl}$	$11.9813478 \pm 0.0003326$	$0.0020 \pm 0.0015$	$0.0697 \pm 0.1173$
<i>V</i>	$7f_0 - f_{Bl}$	$11.9813985 \pm 0.0001463$	$0.0035 \pm 0.0011$	$0.3031 \pm 0.0516$
<i>I<sub>c</sub></i>	$7f_0$	$11.9980579 \pm 0.0000744$	$0.0052 \pm 0.0009$	$0.6501 \pm 0.0262$
<i>V</i>	$7f_0$	$11.998109 \pm 0.0000852$	$0.0061 \pm 0.0011$	$0.8038 \pm 0.0300$
<i>B</i>	$7f_0$	$11.998197 \pm 0.0000647$	$0.0105 \pm 0.0015$	$0.7772 \pm 0.0228$
<i>V</i>	$7f_0 + f_{Bl}$	$12.0143063 \pm 0.0001302$	$0.0040 \pm 0.0011$	$0.4964 \pm 0.0459$
<i>B</i>	$7f_0 + f_{Bl}$	$12.0150643 \pm 0.0001462$	$0.0046 \pm 0.0015$	$0.4920 \pm 0.0515$
<i>V</i>	$8f_0 - f_{Bl}$	$13.6968279 \pm 0.0001358$	$0.0038 \pm 0.0011$	$0.8045 \pm 0.0479$
<i>I<sub>c</sub></i>	$8f_0$	$13.7120561 \pm 0.0000838$	$0.0046 \pm 0.0009$	$0.7282 \pm 0.0295$
<i>V</i>	$8f_0$	$13.7120923 \pm 0.0000659$	$0.0079 \pm 0.0011$	$0.6955 \pm 0.0232$
<i>B</i>	$8f_0$	$13.7121617 \pm 0.0000795$	$0.0085 \pm 0.0015$	$0.8051 \pm 0.0280$
<i>V</i>	$8f_0 + f_{Bl}$	$13.7281859 \pm 0.0001367$	$0.0038 \pm 0.0011$	$0.1461 \pm 0.0482$
<i>V</i>	$9f_0$	$15.4263194 \pm 0.0001513$	$0.0034 \pm 0.0011$	$0.9414 \pm 0.053$
<i>V</i>	$10f_0 - f_{Bl}$	$17.1237314 \pm 0.0002627$	$0.0019 \pm 0.0011$	$0.6662 \pm 0.0926$
<i>B</i>	$10f_0$	$17.1401791 \pm 0.0001131$	$0.0060 \pm 0.0015$	$0.7061 \pm 0.0399$
<i>V</i>	$10f_0$	$17.1408044 \pm 0.0001680$	$0.0031 \pm 0.0011$	$0.8639 \pm 0.0592$
<i>V</i>	$11f_0$	$18.8539153 \pm 0.0001160$	$0.0044 \pm 0.0011$	$0.4507 \pm 0.0409$



**Fig. 7.** Phase light curves of DU Mon in the  $B$ ,  $V$ , and  $I_c$  bands in the narrow phase intervals of the Blazhko effect corresponding to the minimum and maximum amplitudes.

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